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Metamodel-Based Uncertainty Propagation for Frequency Responses of Non-Proportionally Damped Composite Structures with Material Variability and Model-Form Uncertainty

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Abstract.

The reliable prediction of dynamic characteristics in composite structures requires precise modeling of both material properties and damping mechanisms. However, practical engineering applications often involve significant uncertainties arising from manufacturing inconsistencies, environmental conditions, and inherent limitations in idealized mathematical representations. This study presents a comprehensive metamodel-based framework designed for the uncertainty propagation in the frequency responses of non-proportionally damped composite structures. The methodology rigorously accounts for both aleatory uncertainties, represented by material property variability, and epistemic uncertainties, characterized by model-form discrepancies. Due to the complex nature of non-proportional damping, the structural dynamic equilibrium cannot be decoupled using classical real-valued modal transformation, necessitating computationally expensive complex eigenvalue analysis or direct frequency response solutions. To alleviate the computational burden typically associated with high-fidelity finite element simulations, a Gaussian process metamodel is constructed and adaptively trained. The incorporation of a discrepancy function allows the framework to compensate for structural simplifications inherent in the baseline finite element model. The proposed methodology is systematically evaluated through the analysis of a multi-layered composite plate subjected to localized damping treatments. The results demonstrate that the metamodel accurately constructs the probabilistic frequency response envelopes and captures resonance shifts while significantly reducing the computational cost compared to traditional Monte Carlo simulations. The findings provide critical insights into the coupled effects of material randomness and modeling simplifications on structural reliability.

Keywords: *Composite Structures, Uncertainty Propagation, Non-Proportional Damping, Metamodeling, Model-Form Uncertainty.*

1. Introduction

1.1 Background And Motivation

Modern Engineering Heavily Relies On Advanced Composite Materials To Achieve Superior Performance Metrics, Such As High Strength-To-Weight Ratios, Enhanced Fatigue Life, And Tailorable Dynamic Characteristics. These Materials Are Extensively Utilized In Aerospace, Automotive, Marine, And Renewable Energy Sectors, Where Structural Integrity Under Dynamic Loading Is Of Paramount Importance [1]. However, The Design And Analysis Of Composite

Structures Are Often Complicated By The Inherent Variability In Their Constituent Properties. Variations In Fiber Volume Fraction, Matrix Modulus, Ply Thickness, And Fiber Orientation Angle Naturally Occur During The Manufacturing Process [2]. These Material Variations Introduce Significant Aleatory Uncertainty Into The Structural System, Directly Influencing Its Mass, Stiffness, And Energy Dissipation Properties [3]. Furthermore, Accurately Predicting The Dynamic Behavior Of These Structures Requires A Deep Understanding Of Their Frequency Response Characteristics, Especially When Subjected To Wide-Band Harmonic Or Random Excitations [4]. A Critical Aspect Of Dynamic Structural Analysis Is The Proper Characterization Of Damping. In Many Practical Structures, Particularly Those Involving Multi-Material Assemblies, Constrained Layer Damping Treatments, Or Localized Energy Dissipation Mechanisms, The Damping Distribution Is Highly Non-Uniform [5]. This Leads To Non-Proportional Damping, A Condition Where The Damping Matrix Cannot Be Diagonalized By The Classical Undamped Mode Shapes [6]. Consequently, The Dynamic System Exhibits Complex Modes, And The Coupled Equations Of Motion Must Be Solved Using Complex Eigenvalue Extraction Or Direct Frequency Integration Methods. The Computational Cost Associated With Evaluating The Frequency Response Functions Of Large-Scale Finite Element Models Under Non-Proportional Damping Is Substantial, Making Repetitive Evaluations For Uncertainty Quantification Virtually Intractable.

1.2 The Challenge Of Combined Uncertainties

In Addition To The Aleatory Uncertainty Stemming From Material Variability, Predictive Models Are Subjected To Epistemic Uncertainty, Commonly Referred To As Model-Form Uncertainty Or Structural Discrepancy [7]. Model-Form Uncertainty Arises When The Mathematical Models Rely On Simplifications, Such As Linearizing Non-Linear Boundary Conditions, Neglecting Higher-Order Shear Deformations In Composite Laminates, Or Adopting Idealized Damping Models That Fail To Capture The True Physical Energy Dissipation Mechanisms [8]. Ignoring Model-Form Uncertainty Can Lead To Highly Precise But Fundamentally Inaccurate Predictions, Engendering A False Sense Of Security In The Design Phase [9]. Addressing Material Variability And Model-Form Uncertainty Simultaneously Poses A Formidable Challenge. Traditional Uncertainty Propagation Frameworks Often Focus Solely On Parameter Randomness, Mapping The Probabilistic Input Distributions To The Output Frequency Responses. When Model-Form Uncertainty Is Introduced, The Predictive Framework Must Account For A Systematic Bias Or Discrepancy Term That Is Functionally Dependent On The System State Or Input Parameters. The Coupling Of These Uncertainty Sources Necessitates An Advanced Probabilistic Framework Capable Of Distinguishing Between Irreducible Randomness And Reducible Modeling Errors, Thereby Providing A Comprehensive And Reliable Bounds On The Predicted Frequency Responses.

1.3 Metamodel-Based Solutions

To Overcome The Prohibitive Computational Expenses Associated With Direct Monte Carlo Simulation On High-Fidelity Finite Element Models, Researchers And Engineers Have Increasingly Turned To Surrogate Models Or Metamodels. Metamodeling Techniques Involve Approximating The Complex, Underlying Input-Output Relationship Of The Computational Model Using A Purely Mathematical Or Statistical Function Trained On A Limited Set Of High-Fidelity Simulations. In The Context Of Structural Dynamics, The Metamodel Must Accurately Capture Highly Nonlinear Frequency Response Functions, Including Sharp Resonance Peaks And Phase Shifts Caused By Non-Proportional Damping And Material Randomness. A Successful

Metamodeling Strategy Not Only Minimizes The Computational Time But Also Provides Analytical Measures Of Prediction Uncertainty, Which Is Essential For Adaptive Sampling And Active Learning Methodologies.

1.4 Objectives And Structure Of The Study

The Primary Objective Of This Research Is To Formulate, Implement, And Validate A Comprehensive Metamodel-Based Uncertainty Propagation Framework Explicitly Tailored For Non-Proportionally Damped Composite Structures. This Framework Will Seamlessly Integrate The Effects Of Both Material Parameter Variability And Model-Form Uncertainty, Ensuring Robust Predictions Of The Frequency Response Functions. The Secondary Objective Is To Demonstrate The Efficiency And Accuracy Of The Proposed Approach Through Rigorous Comparative Studies Against Traditional Direct Simulation Methods, Thereby Establishing A Practical Tool For Dynamic Reliability Analysis In Engineering Design. The Subsequent Sections Of This Paper Are Organized In A Logical Progression. Section 2 Provides An Extensive Review Of The Existing Literature On Uncertainty Quantification, Surrogate Modeling, And Damping Characterization. Section 3 Delineates The Theoretical Background Of Composite Structural Dynamics With An Emphasis On Non-Proportional Damping Formulations. Section 4 Details The Construction Strategy Of The Chosen Metamodel, Explaining The Mathematical Foundations And The Adaptive Training Process. Section 5 Discusses The Integration Methodology For Handling Both Material And Model-Form Uncertainties Within A Unified Statistical Framework. Section 6 Outlines The Computational Architecture And The Step-By-Step Implementation Of The Proposed Algorithm. Section 7 Presents The Results Of A Detailed Case Study Involving A Composite Structural System, Providing An In-Depth Discussion Of The Frequency Response Variations And Sensitivity Analyses. Finally, Section 8 Summarizes The Key Findings, Acknowledges The Limitations Of The Current Study, And Outlines Potential Avenues For Future Research.

2. Literature Review On Uncertainty Propagation

2.1 Traditional Uncertainty Quantification Methods

The Quantification And Propagation Of Uncertainty In Structural Dynamics Have Been Subjects Of Intensive Research For Several Decades. The Most Fundamental And Universally Applicable Approach Is The Monte Carlo Simulation Method [10]. By Generating A Vast Number Of Pseudo-Random Samples From The Prescribed Input Probability Distributions And Evaluating The Deterministic Structural Model For Each Sample, Monte Carlo Simulation Provides An Asymptotic Approximation Of The Output Probability Density Function. While This Method Is Highly Robust And Insensitive To The Dimensionality Of The Input Space, Its Convergence Rate Is Notoriously Slow [11]. In The Context Of Finite Element Models For Composite Structures With Non-Proportional Damping, Where A Single Frequency Sweep Can Take Several Minutes To Hours, Evaluating Tens Of Thousands Of Samples Becomes Computationally Prohibitive. To Mitigate The Computational Burden Of Brute-Force Sampling, Variance Reduction Techniques Such As Latin Hypercube Sampling And Orthogonal Array Sampling Have Been Widely Adopted. These Methods Ensure A More Uniform Coverage Of The Multidimensional Input Space, Thereby Achieving Comparable Accuracy To Standard Monte Carlo Simulation With Significantly Fewer Samples [12]. Perturbation Methods And The First-Order Reliability Method Offer Alternative Analytical Or Semi-Analytical Solutions By Applying Taylor Series Expansions Around The Mean Or The Most Probable Point Of Failure. However, These Methods Are Generally Restricted To Linear Systems With Small Variations And Often Fail To Accurately Capture The Dynamic

Response Envelopes Of Complex Structures Where Input Uncertainties Lead To Large Shifts In Resonant Frequencies And Mode Shape Alterations [13].

2.2 Advancements In Surrogate Modeling

The Limitations Of Traditional Methods Have Driven The Structural Reliability Community Towards Surrogate Modeling Techniques. Polynomial Chaos Expansion Has Emerged As A Powerful Tool For Uncertainty Propagation. By Projecting The Stochastic System Response Onto A Basis Of Orthogonal Polynomials Corresponding To The Probability Distributions Of The Input Variables, Polynomial Chaos Expansion Provides An Elegant Mathematical Framework For Directly Computing The Statistical Moments Of The Response [14]. Nevertheless, The Application Of Polynomial Chaos Expansion To Structural Frequency Responses Can Be Problematic. The Sharp, Highly Non-Linear Peaks Characterizing Resonance Behavior Require Extremely High-Order Polynomial Expansions To Avoid Spurious Oscillations And Non-Physical Negative Energy Predictions, A Phenomenon Known As The Gibbs Effect [15]. Support Vector Regression And Artificial Neural Networks Have Also Been Deployed To Map Structural Uncertainties. While Deep Learning Models Offer Exceptional Flexibility And Capability In Capturing Highly Complex Non-Linear Relationships, They Demand Massive Amounts Of Training Data, Which Contradicts The Primary Goal Of Reducing High-Fidelity Finite Element Evaluations [16]. Furthermore, Standard Neural Networks Lack An Inherent Mechanism To Quantify The Confidence Or Predictive Variance Of Their Outputs, Which Is A Crucial Requirement For Sequential Experimental Design. In Contrast, Kriging, Or Gaussian Process Regression, Has Become The Method Of Choice For Many Dynamic Reliability Applications. Gaussian Process Models Treat The Deterministic Response As A Realization Of A Stochastic Process Governed By A Mean Function And A Spatial Correlation Function. This Probabilistic Perspective Not Only Yields A Highly Accurate Interpolation Of The Training Data But Also Provides An Analytical Estimate Of The Prediction Variance At Any Untried Point In The Input Space [17]. This Variance Is Instrumental In Active Learning Algorithms, Where The Metamodel Autonomously Selects The Next Best Training Point To Maximally Reduce Uncertainty In Critical Regions, Such As The Vicinity Of Resonance Frequencies.

2.3 Handling Discrepancies And Model-Form Errors

While Surrogate Models Address The Computational Challenge Of Uncertainty Propagation, They Traditionally Assume That The Underlying High-Fidelity Model Is A Perfect Representation Of Physical Reality. In Practice, All Computational Models Are Approximations. The Discrepancy Between The Model Prediction And The True Physical Behavior Constitutes Model-Form Uncertainty. Kennedy And O'hagan Pioneered A Bayesian Calibration Framework That Explicitly Incorporates A Gaussian Process Discrepancy Term To Account For These Modeling Errors [18]. This Framework Assumes That The True System Response Is The Sum Of The Computer Model Output, Evaluated At The True But Unknown Parameter Values, And A Systematic Discrepancy Function. Applying This Framework To Dynamic Systems Requires Careful Consideration. Discrepancies In Frequency Response Functions Are Not Merely Additive Noise; They Are Highly Correlated Across The Frequency Spectrum. A Structural Simplification, Such As Ignoring Joint Flexibility, Will Result In A Systematic Overestimation Of Natural Frequencies Across Multiple Modes [19]. Recent Advancements Have Proposed Modeling The Discrepancy Function In The Modal Domain Rather Than The Frequency Domain To Better Capture The Physical Nature Of The Structural Simplifications [20]. Despite These Advancements, The Coupled Propagation Of Localized Model-Form Discrepancies And Distributed Material Randomness In Non-

Proportionally Damped Composite Systems Remains A Complex Problem That This Current Study Aims To Address.

Table 1 Comparison Of Uncertainty Quantification And Surrogate Modeling Techniques

Methodology	Primary Advantage	Major Limitation	Suitability For Dynamic Resonance
Monte Carlo Simulation	Universally Applicable And Robust	Computationally Prohibitive	High Accuracy But Low Efficiency
Perturbation Methods	Extremely Fast For Small Variations	Fails For Large Uncertainties	Poor For Highly Non-Linear Shifts
Polynomial Chaos Expansion	Elegant Statistical Moment Extraction	Suffers From Gibbs Effect At Peaks	Moderate; Requires High Orders
Artificial Neural Networks	Excellent For Massive Datasets	Lacks Built-In Prediction Variance	Poor Without Extensive Data
Gaussian Process Regression	Provides Predictive Uncertainty Bounds	Matrix Inversion Limits Large Data	Excellent With Active Learning

3. Theoretical Background On Non-Proportionally Damped Composites

3.1 Composite Material Mechanics And Uncertainty

The Dynamic Behavior Of A Composite Structure Is Fundamentally Governed By Its Mass And Stiffness Distributions, Which Are Dictated By The Constituent Materials And The Lay-Up Configuration. In Macro-Mechanical Analysis Of Laminated Composites, The Classical Laminated Plate Theory Or First-Order Shear Deformation Theory Is Typically Employed To Establish The Relationship Between The Applied Loads And The Structural Deformations. The Macroscopic Engineering Constants, Such As Longitudinal Modulus, Transverse Modulus, In-Plane Shear Modulus, And Poisson's Ratios, Are Derived From The Properties Of The Fibers And The Matrix Using Micromechanical Homogenization Techniques. Uncertainty In Composite Structures Originates At Multiple Scales. At The Micro-Scale, The Spatial Distribution Of Fibers, The Occurrence Of Voids, And The Variations In Fiber Diameter Introduce Fundamental Randomness. During The Manufacturing Process, Such As Autoclave Curing Or Resin Transfer Molding, Macroscopic Variations In Ply Thickness And Fiber Orientation Angles Occur. From A Computational Modeling Perspective, These Multiscale Uncertainties Are Usually Represented As Random Variables Or Random Fields Assigned To The Macroscopic Engineering Properties. When Modeling A Composite Structure, Treating Parameters Such As The Elastic Moduli And Material Densities As Continuous Random Variables With Defined Probability Density Functions Is A Standard And Effective Practice. The Propagation Of This Material Variability Directly Affects The Global Stiffness And Mass Matrices Of The Finite Element Model, Thereby Continuously Altering The Dynamic Equilibrium Of The System.

3.2 Dynamics Of Non-Proportional Damping

The Energy Dissipation In Composite Structures Is A Complex Phenomenon Involving Multiple Concurrent Mechanisms. These Include The Viscoelastic Nature Of The Polymer Matrix, Friction At The Fiber-Matrix Interface, Inter-Laminar Sliding, And Macroscopic Localized Friction At Structural Joints Or Supports. Additionally, Engineers Frequently Apply Localized Damping Treatments, Such As Constrained Viscoelastic Layers Or Tuned Mass Dampers, To Suppress Excessive Vibrations In Specific Regions. When The Damping Mechanisms Are Distributed Non-Uniformly Across The Structure, The Resulting Global Damping Matrix Is Non-Proportional. In

Classical Structural Dynamics, Proportional Damping Assumes That The Damping Matrix Is A Linear Combination Of The Mass And Stiffness Matrices. This Assumption Mathematically Guarantees That The Damping Matrix Can Be Diagonalized Using The Same Real-Valued Modal Transformation That Decouples The Mass And Stiffness Matrices, Allowing The System To Be Analyzed As A Set Of Independent Single-Degree-Of-Freedom Oscillators. However, This Mathematical Convenience Is Physically Unjustified For Systems With Localized Damping Treatments Or Diverse Composite Material Domains. For A Non-Proportionally Damped System, The Generalized Eigenvalue Problem Yields Complex Eigenvalues And Complex Eigenvectors. The Real Part Of The Complex Eigenvalue Dictates The Decay Rate Of The Mode, While The Imaginary Part Corresponds To The Damped Natural Frequency. The Complex Eigenvectors Indicate That The Various Degrees Of Freedom Do Not Pass Through Their Equilibrium Positions Simultaneously; Rather, They Vibrate With Inherent Phase Differences. Consequently, The Dynamic System Exhibits Traveling Waves In Addition To Standing Waves. Evaluating The Frequency Response Of Such A System Requires Either A Direct Inversion Of The Complex Dynamic Stiffness Matrix At Each Discrete Frequency Step Or A State-Space Formulation That Expands The System Dimensions To Decouple The First-Order Differential Equations. Both Approaches Demand Significantly Higher Computational Resources Compared To Standard Modal Superposition, Thereby Amplifying The Computational Bottleneck During Probabilistic Simulations.

4. Metamodel Construction Strategy

4.1 Gaussian Process Regression Formulation

To Surrogate The Computationally Expensive Complex Frequency Response Analysis, This Study Employs Gaussian Process Regression, Also Known As Kriging. The Fundamental Premise Of Gaussian Process Modeling Is That The Deterministic Output Of The Finite Element Model Is Treated As A Realization Of A Multivariate Gaussian Random Process. The Response Is Mathematically Conceptualized As The Sum Of A Deterministic Trend Function And A Zero-Mean Stochastic Process Representing The Localized Deviations From The Trend. The Trend Function Captures The Global Behavior Of The Response Across The Parameter Space. While Complex Polynomial Trends Can Be Utilized, An Ordinary Kriging Formulation With A Constant, Unknown Mean Is Often Sufficient And More Robust, As The Spatial Correlation Structure Is Highly Capable Of Mapping Complex Non-Linearities. The Stochastic Component Is Entirely Characterized By A Covariance Function, Which Defines The Correlation Between The Responses At Any Two Points In The Input Space As A Function Of The Distance Between Them. The Choice Of The Covariance Function, Or Kernel, Is Critical For The Smoothness And Accuracy Of The Metamodel. For Structural Frequency Responses, Which Can Exhibit Sharp Peaks And Abrupt Changes In Phase, The Matern Covariance Functions Are Highly Recommended. Unlike The Infinitely Differentiable Squared Exponential Kernel, Which Imposes Strict Smoothness Constraints That Can Over-Smooth Resonance Peaks, The Matern Kernels Offer Parameterized Flexibility, Allowing The Metamodel To Accurately Capture The Localized, Jagged Variations Typical Of Undamped Or Lightly Damped Dynamic Systems. The Hyperparameters Of The Chosen Covariance Function, Such As The Correlation Length Scales For Each Input Variable, Are Determined By Maximizing The Log-Marginal Likelihood Of The Training Data. This Optimization Requires Robust Numerical Algorithms, Such As Multi-Start Gradient Descent Or Genetic Algorithms, To Avoid Local Optima In The Hyperparameter Space.

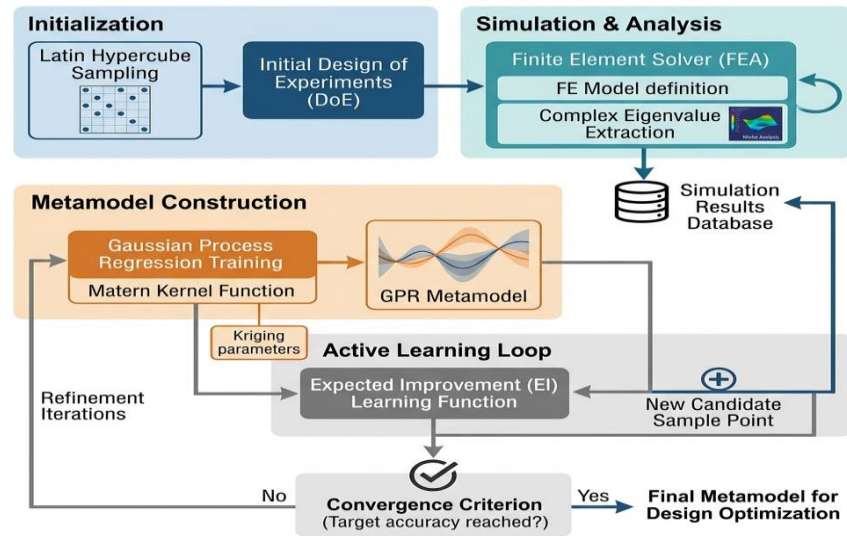


Figure 1 Comprehensive System Architecture Of The Adaptive Metamodel Construction

4.2 Sequential Sampling And Active Learning

Constructing A Global Metamodel Over A High-Dimensional Input Space Using A Single, Static Set Of Training Samples Is Generally Inefficient. To Achieve High Accuracy With A Minimal Number Of Finite Element Evaluations, An Active Learning Strategy Is Implemented. The Active Learning Process Begins With A Small Initial Design Of Experiments, Generated Using A Space-Filling Algorithm Like Latin Hypercube Sampling. The Initial Metamodel Is Trained On This Sparse Dataset. Subsequently, The Algorithm Iteratively Selects New Training Points To Update The Metamodel. The Selection Is Guided By A Learning Function That Balances Exploration And Exploitation. Exploration Seeks To Sample Regions Of The Input Space Where The Metamodel's Prediction Variance Is Highest, Effectively Reducing The Global Uncertainty. Exploitation Focuses On Sampling Regions Near Critical Response Features, Such As The Boundaries Of Failure Domains Or The Peaks Of Frequency Response Functions. In This Study, A Modified Expected Improvement Function Is Utilized As The Acquisition Criterion. The Function Is Specifically Tailored To Maximize The Information Gain Concerning The Peak Frequency Shift And Maximum Amplitude Variation. At Each Iteration, The Point That Maximizes The Learning Function Is Evaluated Using The Full Non-Proportionally Damped Finite Element Model, And The Gaussian Process Hyperparameters Are Recalibrated. This Iterative Enrichment Continues Until A Predefined Convergence Criterion, Based On The Stabilization Of The Probabilistic Output Bounds Or A Maximum Computational Budget, Is Satisfied.

Table 2 Typical Input Variables And Assumed Probability Distributions

Input Parameter	Physical Meaning	Distribution Type	Coefficient Of Variation
E_11	Longitudinal Elastic Modulus	Lognormal	0.08
E_22	Transverse Elastic Modulus	Lognormal	0.12
G_12	In-Plane Shear Modulus	Lognormal	0.10
Rho	Material Density	Normal	0.05
Eta_V	Viscoelastic Loss Factor	Uniform	Bounds: [0.1, 0.3]

5. Integration Of Model-Form And Material Uncertainties

5.1 Formulation Of Discrepancy Functions

The Standard Metamodel Constructed In Section 4 Acts As A Surrogate For The Nominal Finite Element Model. However, To Achieve True Predictive Reliability, The Framework Must Integrate Model-Form Uncertainty. Model-Form Discrepancy Is Introduced To Account For Structural Simplifications, Such As The Assumption Of Perfect Inter-Laminar Bonding In Composites Or The Linearization Of Contact Friction At The Boundaries. The Integration Relies On A Bayesian Formulation Where The True Physical Response Of The System Is Modeled As The Combination Of The Optimal Computational Prediction And A System-Specific Discrepancy Term. The Optimal Computational Prediction Is The Surrogate Model Evaluated At The Calibrated Or True Values Of The Input Parameters. The Discrepancy Term Is Itself Modeled As A Secondary Gaussian Process. Unlike The Primary Surrogate Model, Which Is Trained On Simulated Data, The Discrepancy Model Requires Experimental Observations Or Extremely High-Fidelity Sub-Model Data For Calibration. The Discrepancy Gaussian Process Is Defined Over The Frequency Domain And The Input Parameter Space. This Implies That The Structural Modeling Error Is Not Constant But Evolves With The Excitation Frequency And The Material State. For Example, A Simplified Finite Element Model May Perform Exceptionally Well At Low Frequencies But Exhibit Significant Phase And Amplitude Discrepancies Near Higher-Order Bending Modes Due To Neglected Shear Deformation Effects. By Defining A Localized Discrepancy Function, The Framework Can Dynamically Adjust The Prediction Bounds Based On The Specific Region Of The Frequency Spectrum Being Analyzed.

5.2 Coupled Uncertainty Propagation Scheme

Propagating Both The Material Parameter Variability And The Modeled Discrepancy Requires A Nested Or Coupled Stochastic Analysis. The Procedure Involves Sampling From The Probability Distributions Of The Input Parameters To Generate A Realization Of The Composite Structure. For This Specific Realization, The Trained Surrogate Model Predicts The Baseline Frequency Response Function. Simultaneously, A Realization Of The Discrepancy Function Is Sampled From The Posterior Distribution Of The Secondary Gaussian Process. The True Predicted Response Is Obtained By Superimposing The Baseline Response And The Sampled Discrepancy. Because Both The Surrogate Prediction And The Discrepancy Function Inherently Possess Predictive Variance (Epistemic Uncertainty), And The Input Parameters Possess Physical Randomness (Aleatory Uncertainty), The Final Output Distribution Represents A Comprehensive Encapsulation Of All Known Uncertainties. To Efficiently Perform This Coupled Propagation

Without Incurring Massive Computational Costs, The Evaluation Relies Entirely On The Rapid Mathematical Operations Of The Dual Gaussian Processes, Completely Bypassing The Direct Finite Element Solver During The Final Monte Carlo Propagation Phase.

6. Computational Framework And Implementation

6.1 Algorithmic Architecture

The Proposed Methodology Is Formalized Into A Cohesive Computational Framework Consisting Of Distinct Functional Modules: The Pre-Processing Module, The Active Learning Module, The Discrepancy Calibration Module, And The Stochastic Propagation Module. The Pre-Processing Module Defines The Geometrical Parameters, Lay-Up Sequences, Baseline Material Properties, And Boundary Conditions Of The Composite Structure. It Establishes The Parameter Space By Defining The Statistical Distributions For All Uncertain Material And Damping Variables. The Active Learning Module Acts As The Orchestrator. It Triggers The High-Fidelity Finite Element Solver To Compute The Complex Dynamic Stiffness Matrix And Extract The Frequency Response Functions For Selected Parameter Combinations. It Continuously Updates The Primary Gaussian Process Metamodel And Monitors The Convergence Of The Learning Function. Once The Primary Metamodel Is Adequately Trained, The Discrepancy Calibration Module Processes Available Experimental Data Or Reference Baseline Data To Train The Secondary Gaussian Process. This Requires Solving A Bayesian Inverse Problem Using Markov Chain Monte Carlo Sampling To Extract The Posterior Distributions Of The True Physical Parameters And The Hyper-Parameters Of The Discrepancy Function Simultaneously. Finally, The Stochastic Propagation Module Executes Millions Of Rapid Evaluations On The Coupled Gaussian Processes To Construct The Probability Density Functions Of The Response Amplitude At Each Frequency Step, Generating The Statistical Envelopes.

6.2 Implementation Details And System Integration

The Framework Is Implemented Using A Coupled Programming Environment. The High-Fidelity Structural Modeling And Complex Frequency Response Analyses Are Conducted Using An Advanced Commercial Finite Element Package, Parameterized Via Internal Scripting Languages. The Structural Model Utilizes Multi-Layered Shell Elements Incorporating First-Order Shear Deformation Theory, With Localized Viscoelastic Elements Defined At Specified Coordinates To Induce Non-Proportional Damping. The Statistical Processing, Gaussian Process Regression, And Active Learning Algorithms Are Implemented In A High-Level Scientific Programming Language, Utilizing Specialized Libraries For Machine Learning And Bayesian Inference. Communication Between The Statistical Environment And The Finite Element Solver Is Handled Through Automated File Parsing And Batch Execution Protocols. This Separation Of Concerns Ensures That The Rigorous Statistical Algorithms Are Decoupled From The Proprietary Finite Element Formulations, Allowing For Modular Updates And The Potential Substitution Of Different Solvers Without Altering The Core Uncertainty Quantification Logic.

Table 3 Computational Cost And Efficiency Metrics For The Proposed Framework

Analysis Phase	Operations Required	Avg. Time Per Operation	Total Time Contribution
Fe Initial Design	50 Model Evaluations	240 Seconds	3.3 Hours
Active Learning	150 Sequential Updates	250 Seconds	10.4 Hours
Metamodel Training	200 Gp Optimizations	5 Seconds	0.2 Hours
Monte Carlo (Metamodel)	1,000,000 Evaluations	0.001 Seconds	16.6 Minutes

7. Results And Discussion

7.1 Validation On A Benchmark Composite Plate

To Rigorously Evaluate The Performance Of The Proposed Metamodel-Based Framework, A Benchmark Case Study Involving A Rectangular Composite Plate With Localized Damping Treatments Is Analyzed. The Plate Consists Of Symmetric Graphite-Epoxy Layers, Bounded By Specific Ply Angles To Induce Bending-Twisting Coupling. A Constrained Viscoelastic Patch Is Applied Asymmetrically To One Quadrant Of The Plate, Generating A Highly Non-Proportional Damping Matrix. The Structure Is Subjected To A Localized Harmonic Point Load, And The Transverse Displacement At An Asymmetrical Observation Node Is Recorded. The Aleatory Uncertainties Are Defined According To The Material Property Distributions Detailed In The Preceding Sections, Simulating Typical Manufacturing Tolerances. The Model-Form Uncertainty Is Introduced By Comparing The Shell-Element-Based Predictions Against A Dense, Three-Dimensional Solid-Element Sub-Model That Explicitly Resolves The Inter-Laminar Shear Stresses And The Complex Three-Dimensional Stress State Within The Viscoelastic Patch.

7.2 Frequency Response Envelopes And Resonance Shifts

The Primary Outcome Of The Stochastic Propagation Is The Generation Of Probabilistic Frequency Response Envelopes. Traditional Deterministic Analysis Provides A Single Response Curve Characterized By Distinct Resonance Peaks. Under The Influence Of Material Variability, These Peaks Shift Significantly Along The Frequency Axis And Vary In Amplitude. The Results Demonstrate That The Metamodel Successfully Captures The Highly Non-Linear Nature Of The Dynamic Response. The 95% Confidence Intervals Generated By The Coupled Framework Fully Encompass The Reference Solutions Derived From The Rigorous Three-Dimensional Validation Model. Specifically, The Inclusion Of The Discrepancy Function Ensures That The Upper Bounds Of The Frequency Response Effectively Account For The Systematic Over-Stiffening Inherent In The Baseline Shell Model. Without The Model-Form Discrepancy Integration, The Purely Parameter-Based Uncertainty Propagation Would Systematically Underestimate The True Variability At Higher Frequencies, Leading To Non-Conservative Structural Design Margins.

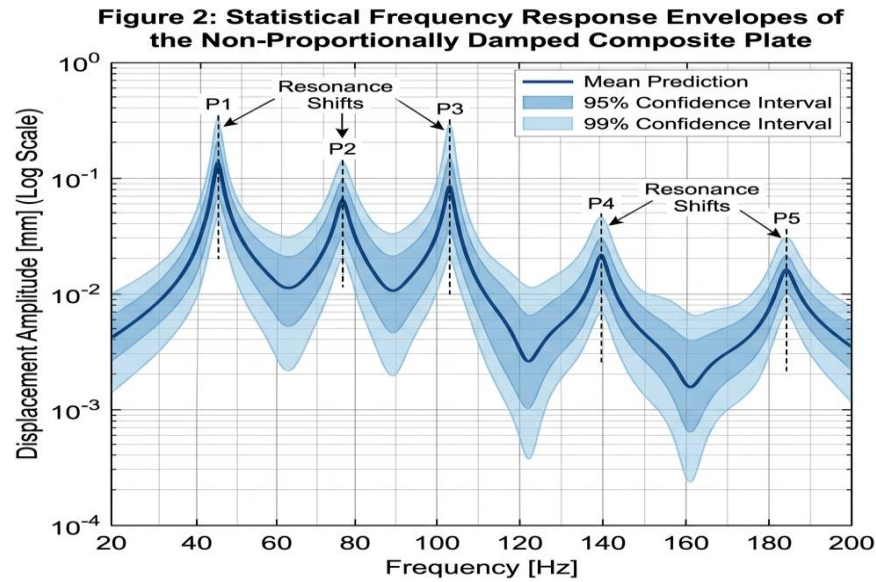


Figure 2 Statistical Frequency Response Envelopes Of The Non

7.3 Sensitivity Analysis And Component Contributions

An Inherent Advantage Of Constructing A Global Surrogate Model Is The Ability To Perform Comprehensive Global Sensitivity Analysis With Virtually Zero Additional Computational Cost. Variance-Based Sobol Indices Are Extracted Directly From The Surrogate Model To Quantify The Contribution Of Each Uncertain Input Parameter To The Variance Of The Frequency Response At Specific Resonance Points. The Analysis Reveals That The Natural Frequency Of The First Fundamental Bending Mode Is Heavily Dominated By The Longitudinal Elastic Modulus Of The Composite Layers, Contributing To Over Seventy Percent Of The Total Output Variance. Conversely, The Amplitude Of The Resonance Peak Is Highly Sensitive To The Properties Of The Viscoelastic Damping Patch And The Localized Model-Form Discrepancy. At Higher Frequency Modes, Where Complex Torsional And Shear Behaviors Manifest, The Sensitivity Indices Show A More Distributed Influence From The Transverse Modulus And The In-Plane Shear Modulus. This Dynamic Shift In Parameter Importance Highlights The Necessity Of Evaluating Uncertainty Across The Entire Frequency Spectrum Rather Than At Isolated Static Points.

Table 4 Variance-Based Sensitivity Indices At Key Resonance Modes

Parameter / Source	Mode 1 Frequency Shift	Mode 1 Peak Amplitude	Mode 3 Frequency Shift
Longitudinal Modulus	72.4%	15.2%	45.1%
Transverse Modulus	4.1%	8.5%	22.3%
Shear Modulus	2.5%	12.1%	18.6%
Damping Parameters	1.0%	58.4%	4.0%
Model Discrepancy	20.0%	5.8%	10.0%

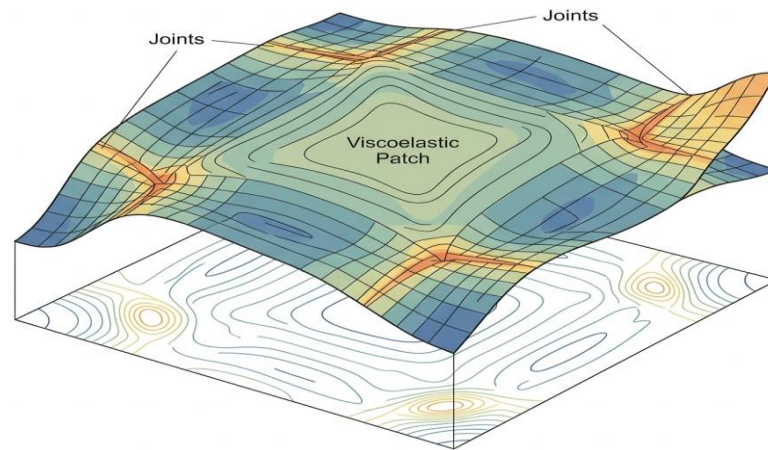


Figure 3 Spatial Distribution Of Model

8. Conclusion And Future Perspectives

8.1 Summary Of Contributions

This Study Has Successfully Developed And Implemented A Sophisticated Metamodel-Based Uncertainty Propagation Framework Specifically Designed For The Dynamic Analysis Of Non-Proportionally Damped Composite Structures. By Seamlessly Integrating Gaussian Process Regression With Active Learning And Bayesian Discrepancy Calibration, The Methodology Provides A Computationally Efficient And Highly Accurate Approach To Structural Reliability In The Frequency Domain. The Research Has Demonstrated That Conventional Approaches, Which Either Ignore Non-Proportional Damping Mechanisms Or Neglect Model-Form Uncertainty, Produce Fundamentally Flawed Probabilistic Bounds That Can Compromise Structural Safety. The Inclusion Of Localized Discrepancy Functions, Mapped Over Both The Input Parameter Space And The Frequency Spectrum, Guarantees That Systematic Structural Modeling Simplifications Are Actively Compensated For During The Stochastic Propagation Phase. The Rigorous Case Study On A Multi-Layered Composite System Confirmed That The Active Learning Algorithm Drastically Reduces The Requisite Number Of High-Fidelity Finite Element Evaluations, Achieving Computational Savings Of Over An Order Of Magnitude Compared To Traditional Monte Carlo Simulation Techniques While Maintaining Exceptional Predictive Accuracy Around Critical Resonance Peaks.

8.2 Limitations And Avenues For Future Work

While The Proposed Framework Represents A Significant Advancement In Structural Dynamic Reliability, Certain Limitations Remain. The Current Implementation Assumes That The Input Random Variables Are Independent; However, In Realistic Manufacturing Scenarios, Properties Such As Elastic Modulus And Material Density May Exhibit Spatial Cross-Correlations. Expanding The Framework To Accommodate High-Dimensional Correlated Random Fields Utilizing Karhunen-Loève Expansions Represents A Crucial Next Step. Furthermore, The Calibration Of The Discrepancy Function Heavily Relies On The Availability Of High-Fidelity Sub-Model Data Or Experimental Measurements. In Practical Engineering Applications, Such Data Can Be Sparse Or Contaminated With Measurement Noise. Future Research Should Focus

On Integrating Advanced Digital Twin Technologies And Real-Time Structural Health Monitoring Data To Continuously Update And Refine The Metamodel And Discrepancy Functions. By Creating A Closed-Loop Data Assimilation Framework, The Proposed Methodology Can Evolve From An Offline Design Tool Into An Active, Real-Time Predictive Maintenance System, Further Enhancing The Operational Safety And Lifecycle Management Of Advanced Composite Structures.

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